

Chapter 7 Volume of Pyramids Cones Spheres and Capacity

Teacher Guide and Diagnostic Answer Key

Teacher Guide

Teacher Chapter-at-a-Glance

A cone/pyramid model is helpful
Pacing
Talk Prompt

Mastery Decision Guide

A missed Tripwire does not automatically mean “redo the chapter.” Confirm the likely misconception, reteach the smallest prerequisite or representation that addresses it, and then use a fresh item before moving on.

Missed Tripwire #1 → The student is using diameter as radius and slant height as vertical height. Next move: Label $r=d\div2$ and mark the perpendicular vertical height before substituting.

Confirmation and Fresh Recheck

Confirm with: Ask the student to label radius, vertical height, and the solid family before selecting the formula.

Fresh recheck: Use a cone with both vertical and slant height shown, then a sphere with diameter given.

Decision rule: Move forward only when the fresh response is secure and the reasoning survives the changed representation or context.

Diagram Production Notes — Chapter 7

Use the diagram master IDs:

Ch18-D01 — Prism, Pyramid, Cylinder, Cone, Sphere Comparison

Ch18-D02 — Pyramid Compared to Matching Prism

Ch18-D03 — Cone Compared to Matching Cylinder

Ch18-D04 — Square Pyramid Volume

Ch18-D05 — Cone Volume

Ch18-D06 — Cone Diameter Given

Ch18-D07 — Sphere Radius

Ch18-D08 — Capacity Conversion

Ch18-D09 — Slant Height Trap

The pyramid and cone diagrams must clearly show vertical height as a straight perpendicular measure. Slant height must be labelled separately and not placed where it can be mistaken for vertical height. The sphere diagram must label radius from centre to surface. The capacity conversion diagram should connect cm^3 , mL, and L cleanly.

Ready to Learn Check

Quick check before starting. A wrong response below points to a prerequisite worth confirming before the chapter builds on it.

1. What is the difference between vertical height and slant height?

Correct answer: Vertical height is perpendicular to the base; slant height runs along the outside.

Likely wrong answer: They are the same.

Possible misconception: Height types are being confused.

Next move: Draw the right-angle marker from apex to base and contrast it with the slanted edge.

2. What is the radius of a sphere with diameter 18 cm?

Correct answer: 9 cm

Likely wrong answer: 18 cm

Possible misconception: Diameter is being used as radius.

Next move: Divide the diameter by 2 before using a radius-based formula.

3. What does volume measure?

Correct answer: Space inside a solid.

Likely wrong answer: Outside area.

Possible misconception: Volume meaning is not secure.

Next move: Use the fill-versus-wrap distinction.

4. How many millilitres are in 1 L?

Correct answer: 1000 mL

Likely wrong answer: 100 mL or 10 000 mL

Possible misconception: Litre-to-millilitre conversion is not secure.

Next move: Use $1 \text{ L} = 1000 \text{ mL}$ as the anchor and scale from it.

Ready-to-Learn item 1

Correct answer: Vertical height is perpendicular to the base; slant height runs along the outside.

Likely wrong answer: They are the same.

Misconception: Height types are being confused.

Next move: Draw the right-angle marker from apex to base and contrast it with the slanted edge.

Ready-to-Learn item 2

Correct answer: 9 cm

Likely wrong answer: 18 cm

Misconception: Diameter is being used as radius.

Next move: Divide the diameter by 2 before using a radius-based formula.

Ready-to-Learn item 3

Correct answer: Space inside a solid.

Likely wrong answer: Outside area.

Misconception: Volume meaning is not secure.

Next move: Use the fill-versus-wrap distinction.

Ready-to-Learn item 4

Correct answer: 1000 mL

Likely wrong answer: 100 mL or 10 000 mL

Misconception: Litre-to-millilitre conversion is not secure.

Next move: Use $1 \text{ L} = 1000 \text{ mL}$ as the anchor and scale from it.

Practice

Assign the full set or select items for targeted review. Teacher solutions follow each question.

1. square pyramid: base side 6 cm, vertical height 9 cm

Solution: 1. Base area: $B = 6 \times 6 = 36$ $V = (36) \times (9) \div 3$ $V = 108$ 108 cm^3

2. square pyramid: base side 10 m, vertical height 12 m

Solution: 2. Base area: $B = 10 \times 10 = 100$ $V = (100) \times (12) \div 3$ $V = 1200 \div 3$ $V = 400$ 400 m^3

3. rectangular pyramid: base 8 cm by 5 cm, vertical height 6 cm

Solution: $3.5 \text{ L} = 3500 \text{ mL}$

4. triangular pyramid: triangular base 10 m by perpendicular height 6 m, pyramid vertical height 9 m Find the volume.

Solution: 4. Triangular base area: $B = (10 \times 6) \div 2$ $B = 60 \div 2$ $B = 30$ $V = (30) \times (9) \div 3$ $V = 90$ 90 m^3

5. radius 3 cm, vertical height 8 cm

Solution: 5. $V = (3.14) \times (3^2) \times (8) \div 3$ $V = (3.14) \times (9) \times (8) \div 3$ $V = 226.08 \div 3$ $V = 75.36$

6. radius 5 m, vertical height 12 m

Solution: 6. $V = (3.14) \times (5^2) \times (12) \div 3$ $V = (3.14) \times (25) \times (12) \div 3$ $V = 942 \div 3$ $V = 314$ 314 m^3

7. diameter 10 cm, vertical height 9 cm

Solution: 7. $r = 10 \div 2 = 5$ $V = (3.14) \times (5^2) \times (9) \div 3$ $V = (3.14) \times (25) \times (9) \div 3$ $V = 706.5 \div 3$ $V = 235.5$

8. diameter 14 m, vertical height 6 m Find the volume.

Solution: 8. $r = 14 \div 2 = 7$ $V = (3.14) \times (7^2) \times (6) \div 3$ $V = (3.14) \times (49) \times (6) \div 3$ $V = 923.16 \div 3$ $V = 307.72$

9. radius 3 cm

Solution: 9. $V = (4 (3.14) \times (3^3)) \div 3$ $V = (4 (3.14) \times (27)) \div 3$ $V = 339.12 \div 3$ $V = 113.04$

10. radius 6 m

Solution: 10. $V = (4 (3.14) \times (6^3)) \div 3$ $V = (4 (3.14) \times (216)) \div 3$ $V = 2712.96 \div 3$ $V = 904.32$

11. diameter 10 cm

Solution: 11. $r = 10 \div 2 = 5$ $V = (4 (3.14) \times (5^3)) \div 3$ $V = (4 (3.14) \times (125)) \div 3$ $V = 1570 \div 3$ $V \approx 523.33$

12. diameter 8 m

Solution: 12. $r = 8 \div 2 = 4$ $V = (4 (3.14) \times (4^3)) \div 3$ $V = (4 (3.14) \times (64)) \div 3$ $V = 803.84 \div 3$ $V \approx 267.95$

13. Convert 750 cm^3 to mL.

Solution: 13. $750 \text{ cm}^3 = 750 \text{ mL}$

14. Convert 2000 cm^3 to L.

Solution: 14. $2000 \text{ cm}^3 = 2000 \text{ mL}$ $2000 \text{ mL} = 2 \text{ L}$

15. Convert 3.5 L to mL.

Solution: 15.

16. A container has volume 1250 cm^3 . What is its capacity in litres?

Solution: 16. $1250 \text{ cm}^3 = 1250 \text{ mL}$ $1250 \text{ mL} = 1.25 \text{ L}$

17. A student uses $V = Bh$ for a pyramid. Explain the mistake.

Solution: 17. A pyramid is one-third of a matching prism, so the formula needs $(1) \div (3)$.

18. A student uses slant height instead of vertical height for a cone. Explain the mistake.

Solution: 18. Volume uses vertical height, not slant height. Slant height lies along the outside surface.

19. A sphere has diameter 12 cm. A student uses 12 cm as the radius. Explain the mistake.

Solution: 19. Diameter is the full distance across the sphere. Radius is half the diameter, so the radius is 6 cm.

20. A student says $5000 \text{ cm}^3 = 5000 \text{ L}$. Explain the mistake.

Solution: 20. $1 \text{ L} = 1000 \text{ cm}^3$, so $5000 \text{ cm}^3 = 5 \text{ L}$, not 5000 L.

Chapter Answer Key

Tripwire Check

The student is not correct.

The formula uses radius, not diameter.

The formula also uses vertical height, not slant height.

Radius:

$$r = 14 \div 2$$

$$r = 7$$

Use vertical height:

$$h = 9$$

Now calculate:

$$V = (1/3)\pi r^2 h$$

$$V = (3.14) \times (7^2) \times (9) \div 3$$

$$7^2 = 49$$

$$V = (3.14) \times (49) \times (9) \div 3$$

$$3.14 \times 49 \times 9 = 1384.74$$

$$V = 1384.74 \div 3$$

$$V = 461.58$$

The correct volume is:

$$461.58 \text{ cm}^3$$

Worksheet 1: Guided Practice

1.

a.

$$V = (1 \text{ Bh}) \div 3$$

b.

$$V = 1\pi r^2 h \div 3$$

c.

$$V = 4\pi r^3 \div 3$$

2.

$$B = 36$$

$$V = (36) \times (9) \div 3$$

$$V = 324 \div 3$$

$$V = 108$$

Volume:

$$108 \text{ cm}^3$$

3.

$$r = 5$$

4.

a. 1

b. 1

c. 1

Worksheet 2: Independent Practice

1.

a.

$$B = 4 \times 4 = 16$$

$$V = (16) \times (9) \div 3$$

$$V = 48$$

$$48 \text{ cm}^3$$

b.

$$B = 7 \times 7 = 49$$

$$V = (49) \times (6) \div 3$$

$$V = 98$$

$$98 \text{ m}^3$$

c.

$$B = 9 \times 5 = 45$$

$$V = (45) \times (8) \div 3$$

$$V = 120$$

$$120 \text{ cm}^3$$

2.

a.

$$V = (3.14) \times (2^2) \times (9) \div 3$$

$$V = (3.14) \times (4) \times (9) \div 3$$

$$V = 113.04 \div 3$$

$$V = 37.68$$

$$37.68 \text{ cm}^3$$

b.

$$V = (3.14) \times (4^2) \times (6) \div 3$$

$$V = (3.14) \times (16) \times (6) \div 3$$

$$V = 301.44 \div 3$$

$$V = 100.48$$

$$100.48 \text{ m}^3$$

c.

$$r = 12 \div 2 = 6$$

$$V = (3.14) \times (6^2) \times (5) \div 3$$

$$V = (3.14) \times (36) \times (5) \div 3$$

$$V = 565.2 \div 3$$

$$V = 188.4$$

$$188.4 \text{ cm}^3$$

3.

a.

$$V = (4 (3.14) \times (3^3)) \div 3$$

$$V = (4 (3.14) \times (27)) \div 3$$

$$V = 113.04$$

$$113.04 \text{ cm}^3$$

b.

$$V = (4 (3.14) \times (5^3)) \div 3$$

$$V = (4 (3.14) \times (125)) \div 3$$

$$V \approx 523.33$$

$$523.33 \text{ m}^3$$

c.

$$r = 12 \div 2 = 6$$

$$V = (4 (3.14) \times (6^3)) \div 3$$

$$V = 904.32$$

$$904.32 \text{ cm}^3$$

4.

a. 600 mL

b. 1.5 L

c. 2250 mL

d. 4.5 L

Worksheet 3: Mixed Practice & Application

1.

Base area:

$$B = 5 \times 5 = 25$$

$$V = (25) \times (3) \div 3$$

$$V = 25$$

The volume is:

$$25 \text{ m}^3$$

2.

a.

$$r = 8 \div 2 = 4$$

b.

$$V = (3.14) \times (4^2) \times (15) \div 3$$

$$V = (3.14) \times (16) \times (15) \div 3$$

$$V = 753.6 \div 3$$

$$V = 251.2$$

The volume is:

$$251.2 \text{ cm}^3$$

3.

a.

$$r = 18 \div 2 = 9$$

b.

$$V = (4 (3.14) \times (9^3)) \div 3$$

$$9^3 = 729$$

$$V = (4 (3.14) \times (729)) \div 3$$

$$3.14 \times 729 = 2289.06$$

$$V = (4 \times 2289.06) \div 3$$

$$V = 9156.24 \div 3$$

$$V = 3052.08$$

The volume is:

$$3052.08 \text{ cm}^3$$

4.

a.

$$3750 \text{ cm}^3 = 3750 \text{ mL}$$

b.

$$3750 \text{ mL} = 3.75 \text{ L}$$

5.

The student used slant height. Cone volume uses vertical height.

$$V = (3.14) \times (6^2) \times (8) \div 3$$

$$V = (3.14) \times (36) \times (8) \div 3$$

$$V = 904.32 \div 3$$

$$V = 301.44$$

The correct volume is:

$$301.44 \text{ cm}^3$$

6.

a. Too large.

b. A cone is one-third of a matching cylinder, so using the full cylinder formula gives three times the cone volume.

c.

$$300 \div 3 = 100$$

The cone volume is:

$$100 \text{ cm}^3$$

Diagnostic Key — High-Leverage Items

Tripwire Check

Correct answer: No. Use $r=7$ cm and vertical height $h=9$ cm: $V=(1/3)\pi r^2 h \approx 461.58 \text{ cm}^3$.

Likely wrong answer: Using $r=14$, $h=11$, or both.

Misconception: The student is using diameter as radius and slant height as vertical height.

Next move: Label $r=d \div 2$ and mark the perpendicular vertical height before substituting.

Tripwire Diagnostic

Correct answer: No. Use $r=7$ cm and vertical height $h=9$ cm: $V=(1/3)\pi r^2 h \approx 461.58 \text{ cm}^3$.

Likely wrong answer: Using $r=14$, $h=11$, or both.

Misconception: The student is using diameter as radius and slant height as vertical height.

Next move: Label $r=d \div 2$ and mark the perpendicular vertical height before substituting.

Exit Ticket Diagnostic — High-Leverage Item

Correct answer: $r=4$ cm and $h=6$ cm; $V=(1/3)\pi(4^2)(6)=100.48 \text{ cm}^3$.

Likely wrong answer: Using $r=8$ cm, producing 401.92 cm^3 .

Misconception: Diameter is being substituted as radius in the cone formula.

Next move: Write $d=8$, $r=4$ before substituting, and mark vertical $h=6$.

Variation Lab — Expected Ideas

1. Matching prism/pyramid: pyramid is one-third the volume.
2. Matching cylinder/cone: cone is one-third the volume.
3. Volume uses vertical/perpendicular height, never slant height.

Transfer Check — Suggested Responses

Familiar: $(1/3)(36)(12)=144 \text{ cm}^3$. Altered: $(1/3)(3.14)(16)(9)=150.72 \text{ cm}^3$. Non-routine: $r=6$; $V=(4/3)(3.14)(216)=904.32 \text{ cm}^3$. Repair: $r=7$ and $h=9$; $V=461.58 \text{ cm}^3$.

Exit Ticket — Answers

1. 90 cm^3
2. $r=4$ cm, $h=6$ cm
3. 4.2 L